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PARADOX OF THE BLUNT EDGE OF AN AIRFOIL IN AN UNSTEADY FLOW

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A paradox of the blunt edge of an airfoil in an unsteady ideal flow is established, which states that the solution of the nonlinear problem of unsteady flow around a blunt-edged airfoil subject to strict boundary conditions at this edge is physically meaningless. The paradox is a consequence of the adopted model of the unsteady fluid flow near the blunt edge, which assumes inflection of streamlines. It is established that the solution of the problem by local replacement of the blunt edge by a sharp edge using the hypothesis on the smoothness of streamlines near the trailing edge is physically meaningful.

The effect of the shape of a trailing edge on trailing-edge flow is differently manifested for steady and unsteady flows. In the model of a steady plane flow of an ideal fluid subject to the Kutta–Joukowski postulate, the trailing-edge shape influences only the velocity at this edge (it is equal to zero at a blunt edge and is different from zero at a sharp edge). A different situation arises in the model of an unsteady plane flow. For this model, the solution of the nonlinear initial boundary-value problem depends strongly on the shape of the trailing edge, which changes both the local and integral characteristics of the flow around the airfoil, including velocity circulation.

Most of the theoretical studies of unsteady plane flows were concerned with solving problems of flow around sharp-edged airfoils. In this case, the nonlinear initial boundary-value problem is usually reduced to successive solution of linear boundary-value problems for a number of discrete values of time for which the airfoil position and the shape of the wake vortex are specified.

The unsteady flow around a blunt-edged airfoil has been studied inadequately. This is due primarily to the fact that at a blunt edge, the Kutta–Joukowski postulate is supplemented by the condition of zero relative fluid velocity at the corner point formed by the wake vortex and the airfoil contour. As a result, the flow in a small neighborhood of a blunt edge becomes substantially nonlinear. It should be noted that the well-known algorithms of solution of the relevant initial boundary-value problem ignore this additional condition [1] or require that the consequence of this condition rather than the condition itself be satisfied (see, e.g., [2–5]).

Gorelov and Smolin [6] proposed an algorithm of numerical solution of the nonlinear problem of unsteady flow around a blunt-edged airfoil, in which the indicated condition was strictly satisfied. It was found that strict satisfaction of the condition of zero relative velocity at the corner point formed by the vortex wake and the airfoil contour generates a discontinuous solution for both the local flow characteristics near the trailing edge and for the time derivative of the velocity circulation. This result is inconsistent with the real flow pattern.

The present work is a continuation of the studies initiated in [6]. We performed a more complete numerical experiment using the algorithms designed in [6]. Results of the experiment showed that the adopted ideal flow model leads to a paradox of a blunt edge in an unsteady flow, and it is necessary to improve this model for the neighborhood of a blunt edge.

1. We formulate the main assumptions in the formulation of the nonlinear initial boundary-value problem of unsteady flow around a blunt trailing edge airfoil. The fluid is ideal and incompressible, and the flow outside the airfoil and the wake vortex is potential. The wake vortex results from a change in velocity circulation around the airfoil $\Gamma(t)$ with time t and sheds tangentially to the trailing edge from the upper or lower surface, depending on the sign of the derivative $d\Gamma(t)/dt$ (Fig. 1).

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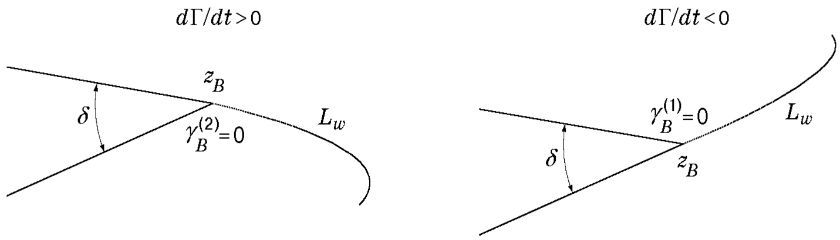

Fig. 1

The wake vortex $L_w(t)$ is simulated by a discontinuity line whose shape for each time is determined by solving the Cauchy problem subject to the conditions that at the initial time $t = 0$, the wake vortex is absent (at $t < 0$, the fluid flow is considered steady). The airfoil contour is simulated by vortex layers.

The intensities of the vortex layers on the upper and lower surfaces of the airfoil at its trailing edge (point z_B) are denoted by $\gamma_B^{(1)}(t)$ and $\gamma_B^{(2)}(t)$, respectively. Then, the intensity of the vortices $\gamma_B(t)$ shedding from the airfoil is equal to $\gamma_B(t) = \gamma_B^{(1)}(t) + \gamma_B^{(2)}(t)$. This intensity is linked to the velocity circulation around the airfoil by the relation

$$\frac{d}{dt}\,\Gamma(t) = -\gamma_B(t)w_B(t), \qquad w_B(t) = \frac{1}{2}(\gamma_B^{(2)}(t) - \gamma_B^{(1)}(t)), \tag{1.1}$$

where $w_B(t)$ is the velocity of the vortex shedding of the airfoil.

Relation (1.1) follows from the Kutta–Joukowski postulate and the Thomson theorem on conservation of velocity circulation along a closed fluid contour.

In [6], the following three versions of additional (to the Kutta–Joukowski postulate) boundary conditions at a blunt trailing edge.

Version 1. The conditions

$$\gamma_B^{(1)} = 0 \quad \text{for} \quad \frac{d}{dt}\,\Gamma < 0, \qquad \gamma_B^{(2)} = 0 \quad \text{for} \quad \frac{d}{dt}\,\Gamma > 0 \tag{1.2}$$

imply that at the corner point formed by the wake vortex and the airfoil contour, the relative fluid velocity is equal to zero (Fig. 1).

Version 2. The following condition is satisfied:

$$w_B(t) = |\gamma_B(t)|/2. \tag{1.3}$$

Condition (1.3) follows from (1.2) and the definition of the velocity of the vortex shedding of the airfoil. Substituting (1.3) into (1.1), we obtain

$$\frac{d}{dt}\,\Gamma(t) = -\frac{1}{2}\,\gamma_B(t)|\gamma_B(t)|. \tag{1.4}$$

We note that in studies of unsteady fluid flow around a blunt trailing edge airfoil, it is common to use relation (1.4) rather than the original condition (1.2).

Version 3. An additional condition at a blunt trailing edge is not specified. This version corresponds to local replacement of a blunt edge by a sharp edge (reversal point).

2. Gorelov and Smolin [6] proposed an algorithm of solution of the nonlinear initial boundary-value problem of unsteady flow around a blunt trailing edge airfoil for the above-mentioned versions of additional boundary conditions at the point z_B. This algorithm allows local flow characteristics at the trailing edge of an airfoil to be determined with high accuracy. In this case, the boundary-value problem that arises in each time step reduces to a combined system of integral equations solved by the panel method. The initial segment of the vortex shedding of the airfoil is simulated by a panel with a linear distribution of vortex-layer intensity, and the remaining part of the wake vortex is replaced by a system of discrete vortices. Strict satisfaction of conditions (1.2) is implemented by using a spline of special form. The system of nonlinear algebraic equations to which the problem is reduced in each time step is solved by an iterative method.

We performed a numerical experiment for Kármán–Trefftz airfoils over a wide range of parameters of the problem. Emphasis was on the estimation of the influence of the angle δ (the angle between tangents to the upper and lower sides of the airfoil at the trailing edge) on flow characteristics. Results of calculation of the quantities γ_B,

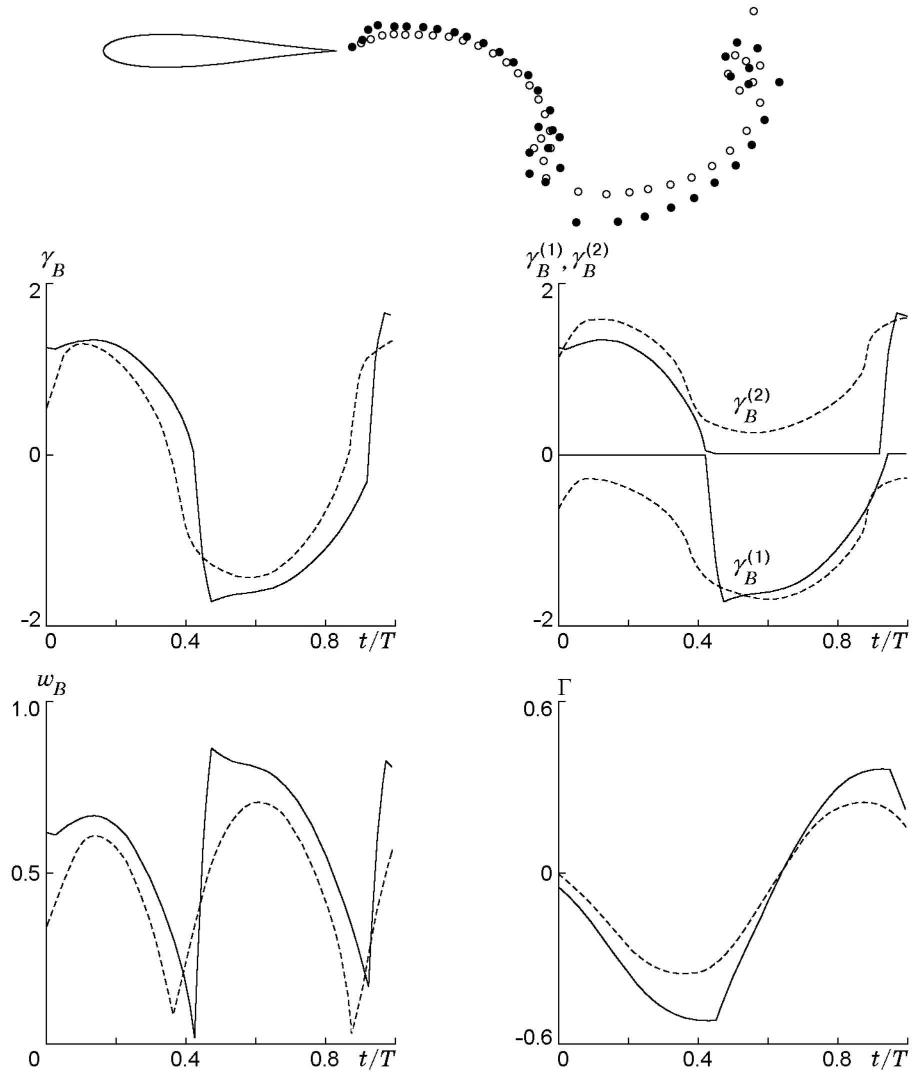

Fig. 2

$\gamma_B^{(1)}$, $\gamma_B^{(2)}$, w_B, and Γ as functions of time t ($0 < t \leqslant T$) for $\delta = 0.1$, 1.0, and 2.5 are given in Figs. 2–4, respectively. The airfoil performed translational vibrations along the normal to the chord with an amplitude of $0.1b$ and Strouhal number $\omega b/v_\infty = \pi$ (b is the airfoil chord, ω is the circular frequency of vibrations, and v_∞ is the velocity of the incident flow). In Figs. 2–4, the calculation results are shown by solid curves for Version 1 and by dashed curves for Version 3. The calculations for Version 2 practically coincide with the calculations for Version 3 and, hence, they are not shown in Figs. 2–4. The airfoils used in the calculations and the shapes of the relevant trailing vortices at the end of the first period are shown at the top of Figs. 2–4, (filled points refer to Version 3 and open points refer to Version 1).

3. We now analyze the results presented in Figs. 2–4. First of all, we consider the singularities of the solution obtained with condition (1.2) satisfied. In the limiting case $\delta \to 0$, it does not become the solution relevant to $\delta = 0$ and is discontinuous for the local flow characteristics at the blunt edge. These statements are illustrated by the data presented in Fig. 2. The discontinuity arises for $t/T = 0.45$, where the time derivative of the velocity circulation vanishes. In the interval $0 < t/T < 0.45$, the derivative $d\Gamma/dt < 0$ and, according to condition (1.2), the intensity of the vortices shedding from the upper surface of the contour $\gamma_B^{(1)}(t)$ is equal to zero. With passage through the point $t/T = 0.45$, the function $\gamma_B^{(1)}(t)$ takes a nonzero value in a jumpwise manner. Similarly, at $0.45 < t/T < 1$, the derivative $d\Gamma/dt > 0$ and the intensity of the vortices shedding from the lower surface of the contour, $\gamma_B^{(2)}(t)$, is equal to zero.

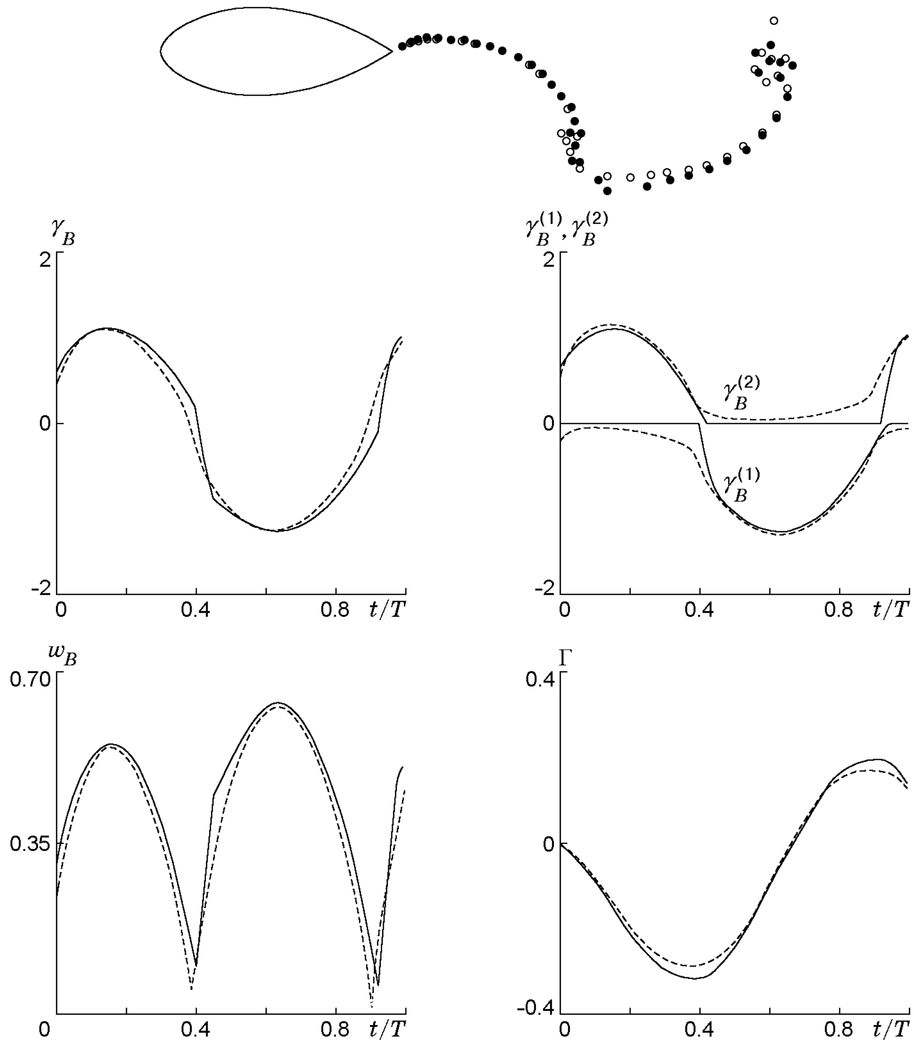

Fig. 3

The solutions obtained for Versions 2 and 3 are different in nature, and as shown by the numerical experiment, they practically do not differ from each other. Boundary condition (1.4) imposes a constraint on the total intensity of trailing-edge vortices γ_B, whereas condition (1.2) is more stringent and requires that the intensity of the vortices shedding from either the upper or lower surfaces of the airfoil be equal to zero. Therefore, condition (1.4) allows us to drop the stringent condition (1.2) and smooth the solution in the neighborhood of the blunt edge. In particular, the limiting case $\delta \to 0$ corresponds to $\delta = 0$. Such solutions were obtained in all papers that considered the problem of unsteady flow around airfoils using condition (1.4).

Thus, condition (1.4) follows from condition (1.2) but the solution obtained in this case does not satisfy the initial condition (1.2). We note that with increase of the angle δ, the difference between these solutions decreases, and at $\delta > 2$, boundary conditions (1.2) and (1.4) generate practically identical solutions (see Fig. 4).

An analysis of results of the numerical experiment leads to the following conclusions.

Strict satisfaction of condition (1.2) leads to a discontinuous solution for the local hydrodynamic characteristics of unsteady fluid flow near the blunt edge, which is inconsistent with the physical flow pattern.

The solution subject to condition (1.2) at the limit $\delta \to 0$ does not becomes the solution of the problem for a sharp-edged airfoil ($\delta = 0$), which, in particular, indicates that the problem of unsteady flow around a blunt-edged airfoil is substantially nonlinear and cannot be linearized at $\delta \ll 1$ within the framework of the flow model considered.

The solution subject to condition (1.4), which follows from condition (1.2), is physically meaningful and practically coincides with the solution of the problem for a sharp-edged airfoil over the entire range of values of the angle δ ($0 < \delta < \pi$).

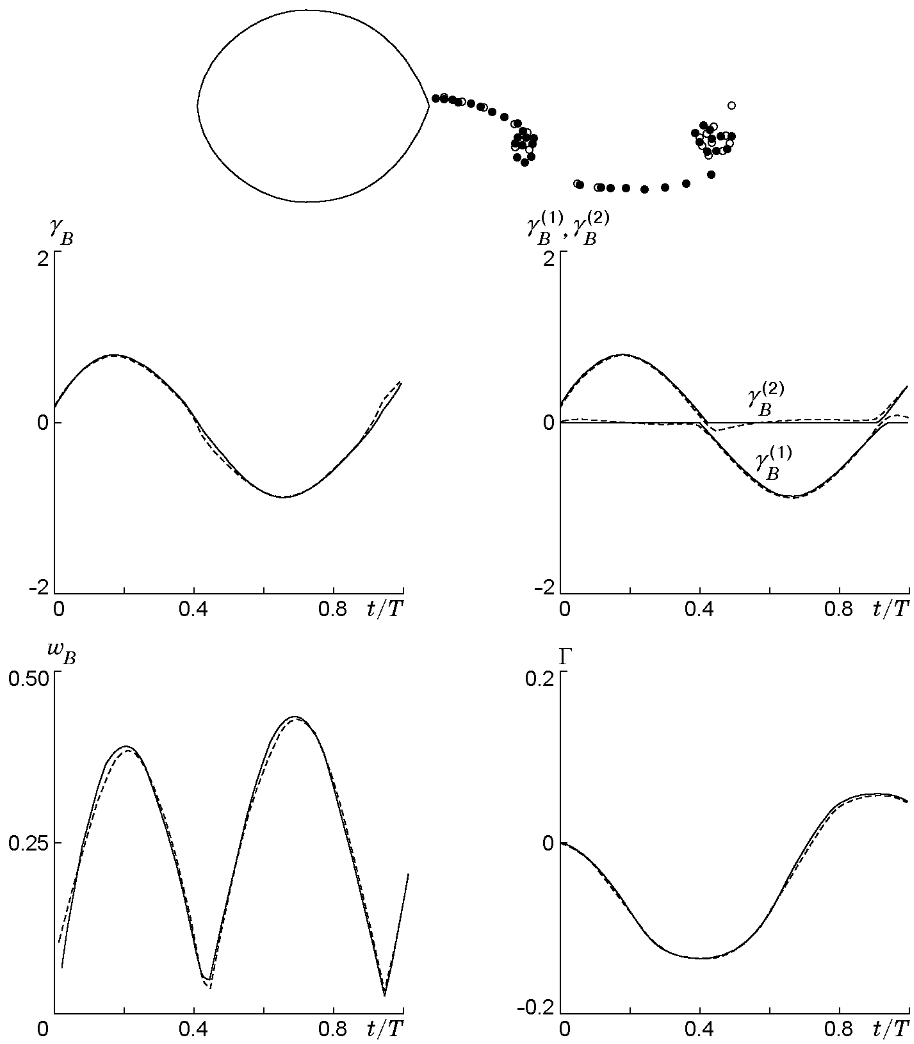

Fig. 4

At the limit $\delta \to \pi$, the solution subject to condition (1.2) coincides with the solution of the problem for a sharp-edged airfoil.

These conclusions are paradoxical in nature and suggest a paradox of the blunt edge of an airfoil in unsteady flow. The essence of this paradox is that the solution of the nonlinear problem of unsteady flow around a blunt-edged airfoil subject to strict boundary conditions in this edge is physically meaningless, whereas the solution of this problem for the same airfoil with local replacement of the blunt edge by a sharp edge is physically meaningful for all values of the angle δ, including $\delta = \pi$.

The paradox is a consequence of the adopted model of unsteady flow in the neighborhood of a blunt edge, which assumes inflection of streamlines. The local replacement of the blunt edge by a sharp edge is equivalent to the hypothesis on the smoothness of streamlines in the neighborhood of the trailing edge. The use of this hypothesis changes the mathematical model of the flow around the airfoil by abandoning the additional condition (1.2). Thus, the Kutta–Joukowski postulate should be used for sharp-edged airfoils.

We note that for steady flows, the local replacement of the blunt edge by a sharp edge is also reasonable. This replacement does not change the velocity circulation around the airfoil but gives nonzero velocity in the trailing edge, which is inherent to real flows. In fact, this flow model takes place in the formulation of the Kutta–Joukowski postulate as the requirement of equal fluid velocities on the upper and lower surfaces of the airfoil with approach to the trailing edge.

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